Relaxation by Modified Logarithmic Barrier Applied to the Problem of Optimal Power Flow DC with Overload

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Initial Motivation

- Numerical problems in Interior Point Methods;
- Overload problems in Optimal Power Flow;

Interior Points Methods

- Path Following Primal-Dual Method
- Classical Logarithmic Barrier;
  - Good results for optimal power flow problems;
  - Numerical problems when very close to the border;
    - Ill-Conditioned;

Model of DC Optimal Power Flow by Network Flow

$$
\begin{aligned}
min \quad & \frac{\alpha}{2} f^t R f + \frac{\beta}{2}(p^t Q p + c^t p) \\
Af \quad &= \quad Ep - l \\
Xf \quad &= \quad 0 \\
f_l \quad &\leq f \leq \quad f_u \\
p_l \quad &\leq p \leq \quad p_u
\end{aligned}
$$

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Model of DC Optimal Power Flow by Network Flow

Change of variables and add slack variables.

$$
\begin{array}{rrcl}
min & \multicolumn{3}{l}{\alpha(\frac{1}{2}f^tRf + c_f^t f) + \beta(\frac{1}{2}p^tQp + c_p^t p)} \\
 & Bf - \widehat{E}p & = & \widehat{l} \\
 & f + s_f & = & f_u \\
 & p + s_p & = & p_u \\
 & (f, p, s_f, s_p) & \geq & 0
\end{array}
$$

$$
B = \begin{bmatrix} A \\ X \end{bmatrix}, \quad \widehat{E} = \begin{bmatrix} E \\ 0 \end{bmatrix}, \quad \widehat{l} = \begin{bmatrix} l^a \\ l^b \end{bmatrix}
$$

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Applying IPM

Lagrangean Function:

$$\mathcal{L} = \varphi(f, p, s_f, s_p) - \mu \sum_{x \in \mathcal{P}} \ln(x_i) - y^t(\widehat{l} - Bf + \widehat{E}p) - w_f^t(f_u - f - s_f) - w_p^t(p_u - p - s_p)$$

$$\varphi(f, p, s_f, s_p) = \alpha(\tfrac{1}{2} f^t R f + c_f^t f) + \beta(\tfrac{1}{2} p^t Q p + c_p^t p)$$

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Applying IPM

Optimality Conditions (KKT):

Primal Feasibility:
$$\begin{cases} Bf - \widehat{E}p &= \widehat{l} \\ f + s_f &= f_u \\ p + s_p &= p_u \end{cases};$$

Dual Feasibility:
$$\begin{cases} B^t y - w_f + z_f - \alpha R f &= \alpha c_f \\ -\widehat{E}^t y - w_p + z_p - \beta Q p &= \beta c_p \end{cases};$$

Complementarity:
$$\begin{cases} F Z_f e &= \mu e \\ P Z_p e &= \mu e \\ S_f W_f e &= \mu e \\ S_p W_p e &= \mu e \end{cases}$$

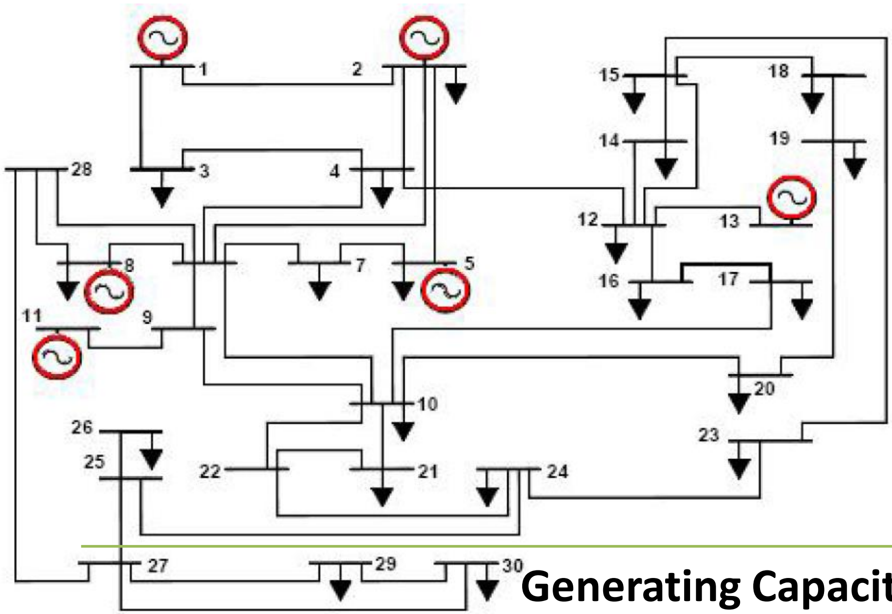
Applying IPM

$$\underbrace{\begin{bmatrix} B & -E & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & I & 0 & 0 & 0 & 0 & 0 \\ -\alpha R & 0 & 0 & 0 & -B^t & I & 0 & -I & 0 \\ 0 & \beta Q & 0 & 0 & -E^t & 0 & I & 0 & -I \\ Z_f & 0 & 0 & 0 & 0 & F & 0 & 0 & 0 \\ 0 & Z_p & 0 & 0 & 0 & 0 & P & 0 & 0 \\ 0 & 0 & W_f & 0 & 0 & 0 & 0 & S_f & 0 \\ 0 & 0 & 0 & W_p & 0 & 0 & 0 & 0 & S_p \end{bmatrix}}_{J_{\mathcal{L}}} . \underbrace{\begin{bmatrix} d_f \\ d_p \\ d_{s_f} \\ d_{s_p} \\ d_y \\ d_{z_f} \\ d_{z_p} \\ d_{w_f} \\ d_{w_p} \end{bmatrix}}_{d} = \underbrace{\begin{bmatrix} r_f \\ r_p \\ r_{s_f} \\ r_{s_p} \\ r_y \\ r_{z_f} \\ r_{z_p} \\ r_{w_f} \\ r_{w_p} \end{bmatrix}}_{r} .$$

Where $J_{\mathcal{L}}$ is a Jacobian Matrix, d is a vector of Newton's directions and r is a residual vector.

Normal Operation

System IEEE 30



Computational Specifications

- Notebook: DELL XPS;
- Ubuntu: 12.4;
- Memory RAM: 8Gb;
- CPU: $Intel^{©}$ $Core^{TM}$ i7-2670QM CPU 2.20GHz × 8;
- Software: Matlab R2012b.

Generating Capacity

1	2	5	8	11	13
10	50	70	70	40	40

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Normal Operation

	Dispatches		
Generator	Tl (MW)	Gc (MW)	Tl & Gc (MW)
1	30.00	30.00	30.00
2	50.00	50.00	50.00
5	70.00	61.70	61.80
8	70.00	61.70	61.60
11	26.15	40.00	40.00
13	37.25	40.00	40.00
	8 iterations	8 iterations	8 iterations

	Lagrange Multipliers w_p		
Generator	Tl (U$/MW)	Gc (U$/MW)	Tl& Gc (U$/MW)
1	7.92×10^{-2}	2.17×10^{2}	2.16×10^{2}
2	4.11×10^{-1}	1.97×10^{2}	1.97×10^{2}
5	9.53×10^{-1}	2.65×10^{-4}	2.61×10^{-4}
8	2.10×10^{-1}	2.65×10^{-4}	2.55×10^{-4}
11	3.47×10^{-8}	168×10^{2}	1.66×10^{2}
13	2.53×10^{-7}	1.68×10^{2}	1.66×10^{2}

Normal Operation

But, how this method behaves in overload situation?

If the demand is greater than the load?

or

If transmission lines are unable to transmit the load?

Overload Situation

- Reducing generator 1: $30MW \rightarrow 10MW$
  - Demand: $283.4MW \rightarrow Capacity\ 280MW$

Generator	Dispatch (MW)	Lagrange Multipliers w_p		
		Tl (U$)	Gc (U$)	Tl & Gc (U$)
1	10.00	1.06×10^{15}	3.79×10^{11}	8.40×10^{14}
2	50.00	1.06×10^{15}	3.79×10^{11}	8.40×10^{14}
5	70.00	1.06×10^{15}	3.79×10^{11}	8.40×10^{14}
8	70.00	1.06×10^{15}	3.79×10^{11}	8.40×10^{14}
11	40.00	1.06×10^{15}	3.79×10^{11}	8.40×10^{14}
13	40.00	1.06×10^{15}	3.79×10^{11}	8.40×10^{14}

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Overload Situation

- Overloads ⇒ Numerical Erros;
- There may be no feasible solution;

- Allows violations in some constraints;
  - In practice, it is possible for a short time;
  - Many cases may be now solved;

Relaxing Constraints

Allow Overloads ⇒ Relax Constraints ;

$$\begin{aligned} s_p &\geq -\delta_p^{-1} \\ s_f &\geq -\delta_f^{-1} \end{aligned}$$

⇒ $p > p_u$ and $f > f_u$ if necessary.

This relaxation does not satisfy the condition for the classical logarithmic barrier;

Modified logarithmic barrier

Proposed by Polyak in LP and nLP;

Minimize: $f(x)$

Subject to: $g_i(x) \geq 0, \quad i = 1, 2, ..., m$

$x \in \Omega = \{x \in R^n | g_i(x) \geq 0\}$

$$\mathcal{M}_{\mathcal{L}}(x,\pi) = \begin{cases} f(x) - \frac{1}{\delta}\sum_{i=1}^{m} \pi_i \ln(\delta g_i(x) + 1) & \text{, se } x \in \Omega_\delta \\ \infty & \text{, se } x \notin \Omega_\delta . \end{cases}$$

$$\Omega_\delta = \{x | \delta g_i(x) + 1 \geq 0\}.$$

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Modified logarithmic barrier

t to infeasible points to Ω, but not to Ω_δ;
ater δ, smaller the expansion Ω_δ;

Existence Condition:

$$g_i(x) \geq -\delta^{-1}$$

- May get to infeasible points to Ω, but not to Ω_δ;
- The greater δ, smaller the expansion Ω_δ;

Applying the Method

- Short Term Operational Capacity;
  - Generator: Maximum Overload 10%;
  - Transmission Line: Maximum Overload 30%;
- It can be viewed as a relaxation operational limits;

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Applying in Optimal Power Flow DC Problem

$$min \quad \alpha(\tfrac{1}{2}f^tRf + c_f^tf) + \beta(\tfrac{1}{2}p^tQp + c_p^tp)$$

$$\begin{aligned} Bf - \hat{E}p &= \hat{l} \\ f + s_f &= f_u \\ p + s_p &= p_u \\ (f, s_f) &\geq -\delta_f^{-1}e_f \\ (p, s_p) &\geq -\delta_p^{-1}e_p. \end{aligned}$$

Lagrangean Function:

$$\mathcal{L} = \varphi(f, p, s_f, s_p) - \delta_x{}^{-1}\sum_{x \in \mathcal{P}} \phi(x, \pi_x, \delta_x) - y^t(\hat{l} - Bf + \hat{E}p) - w_{sf}^t(f_u - f - s_f) - w_{sp}^t(p_u - p - s_p)$$

$$\begin{aligned} \varphi(f, p, s_f, s_p) &= \alpha(\tfrac{1}{2}f^tRf + c_f^tf) + \beta(\tfrac{1}{2}p^tQp + c_p^tp) \\ \phi(x, \pi_x, \delta_x) &= \textstyle\sum_{i \in I} \pi_{xi} ln(\delta_x x_i + 1) \end{aligned}$$

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Applying in Problem of Optimal Power Flow DC

Optimality Conditions

Primal Feasibility

$$\begin{cases} Bf - \hat{E}p &= \hat{l} \\ f + s_f &= f_u \\ p + s_p &= p_u \end{cases}$$

Dual Feasibility

$$\begin{cases} -\alpha Rf + B^t y + z_f - w_{sf} &= \alpha c_f \\ -\beta Qp - \hat{E}^t y + z_p - w_{sp} &= \beta c_p \end{cases}$$

Complementarity

$$\begin{cases} \delta_f Z_f Fe + z_f &= \pi_f \\ \delta_p Z_p Pe + z_p &= \pi_p \\ \delta_f W_{sf} S_f e + w_{sf} &= \pi_{sf} \\ \delta_p W_{sp} S_p e + w_{sp} &= \pi_{sp} \end{cases}$$

$$z_{xi} = \frac{\pi_{xi}}{(\delta_x x_i + 1)}, \; x \in \{f, p\} \text{ e } i \in I$$

Applying in Optimal Power Flow DC Problem

$$\underbrace{\begin{bmatrix} B & -E & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & I & 0 & 0 & 0 & 0 & 0 \\ -\alpha R & 0 & 0 & 0 & -B^t & I & 0 & -I & 0 \\ 0 & \beta Q & 0 & 0 & -E^t & 0 & I & 0 & -I \\ \delta Z_f & 0 & 0 & 0 & 0 & (\delta F + I) & 0 & 0 & 0 \\ 0 & \delta Z_p & 0 & 0 & 0 & 0 & (\delta P + I) & 0 & 0 \\ 0 & 0 & \delta W_f & 0 & 0 & 0 & 0 & (\delta S_f + I) & 0 \\ 0 & 0 & 0 & \delta W_p & 0 & 0 & 0 & 0 & (\delta S_p + I) \end{bmatrix}}_{J_{\mathcal{L}}} \cdot \underbrace{\begin{bmatrix} d_f \\ d_p \\ d_{s_f} \\ d_{s_p} \\ d_y \\ d_{z_f} \\ d_{z_p} \\ d_{w_f} \\ d_{w_p} \end{bmatrix}}_{d} = \underbrace{\begin{bmatrix} r_f \\ r_p \\ r_{s_f} \\ r_{s_p} \\ r_y \\ r_{z_f} \\ r_{z_p} \\ r_{w_f} \\ r_{w_p} \end{bmatrix}}_{r}$$

Where $J_{\mathcal{L}}$ is a Jacobian Matrix, d is a vector of Newton's directions and r is a residual vector.

Normal Operation

Generator	Dispatch		
	Tl (MW)	Gc (MW)	Tl & Gc (MW)
1	30.00	30.00	30.00
2	50.00	50.00	50.00
5	70.00	61.70	61.80
8	70.00	61.70	61.60
11	26.15	40.00	40.00
13	37.25	40.00	40.00
	8 iterations	11 iterations	11 iterations

Before

Generator	Dispatches		
	Tl (MW)	Gc (MW)	Tl & Gc (MW)
1	30.00	30.00	30.00
2	50.00	50.00	50.00
5	70.00	61.70	61.80
8	70.00	61.70	61.60
11	26.15	40.00	40.00
13	37.25	40.00	40.00
	8 iterations	8 iterations	8 iterations

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Normal Operation

	Lagrange Multipliers w_p		
Generator	Tl (U$/MW)	Gc (U$/MW)	Tl & Gc (U$/MW)
1	7.92×10^{-2}	2.17×10^{2}	2.16×10^{2}
2	4.11×10^{-1}	1.97×10^{2}	1.97×10^{2}
5	9.53×10^{-1}	4.61×10^{-8}	4.81×10^{-8}
8	2.10×10^{-1}	4.61×10^{-8}	4.48×10^{-8}
11	2.39×10^{-15}	1.68×10^{2}	1.66×10^{2}
13	5.70×10^{-10}	1.68×10^{2}	1.66×10^{2}

Before

	Lagrange Multipliers w_p		
Generator	Tl (U$/MW)	Gc (U$/MW)	Tl& Gc (U$/MW)
1	7.92×10^{-2}	2.17×10^{2}	2.16×10^{2}
2	4.11×10^{-1}	1.97×10^{2}	1.97×10^{2}
5	9.53×10^{-1}	2.65×10^{-4}	2.61×10^{-4}
8	2.10×10^{-1}	2.65×10^{-4}	2.55×10^{-4}
11	3.47×10^{-8}	168×10^{2}	1.66×10^{2}
13	2.53×10^{-7}	1.68×10^{2}	1.66×10^{2}

Overload Situation

- Reducing generator 1: $30MW \rightarrow 10MW$
  - Demand: $283.4MW \rightarrow Capacity\ 280MW$

- Maximum Overloads:
  - in each Generator: 10%;
  - In each Transmission Line: 30%.

Overload Situation - Generation

	Dispatch					
Generator	Tl (MW)	Overload(%)	Gc (MW)	Overload(%)	Tl & Gc (MW)	Overload(%)
1	10.69	6.86	10.02	0.17	10.00	0.00
2	50.45	0.91	50.02	0.03	50.00	0.00
5	70.27	0.38	69.77	0.00	71.80	2.57
8	70.73	1.05	69.77	0.00	71.59	2.28
11	40.36	0.91	41.82	4.55	40.01	0.01
13	40.90	2.24	42.00	5.00	40.00	0.00
	8 iterations		23 iterations		21 iterations	

	Lagrange Multipliers w_p		
Generator	Tl (U$/MW)	Gc (U$/MW)	Tl & Gc (U$/MW)
1	4.69	1.69×10^2	2.77×10^2
2	4.78	2.29×10^2	2.37×10^2
5	5.27	2.61×10^{-4}	1.00×10^{-3}
8	4.43	2.97×10^{-4}	2.79×10^{-4}
11	3.74	1.95×10^2	2.06×10^2
13	4.09	1.95×10^2	2.06×10^2

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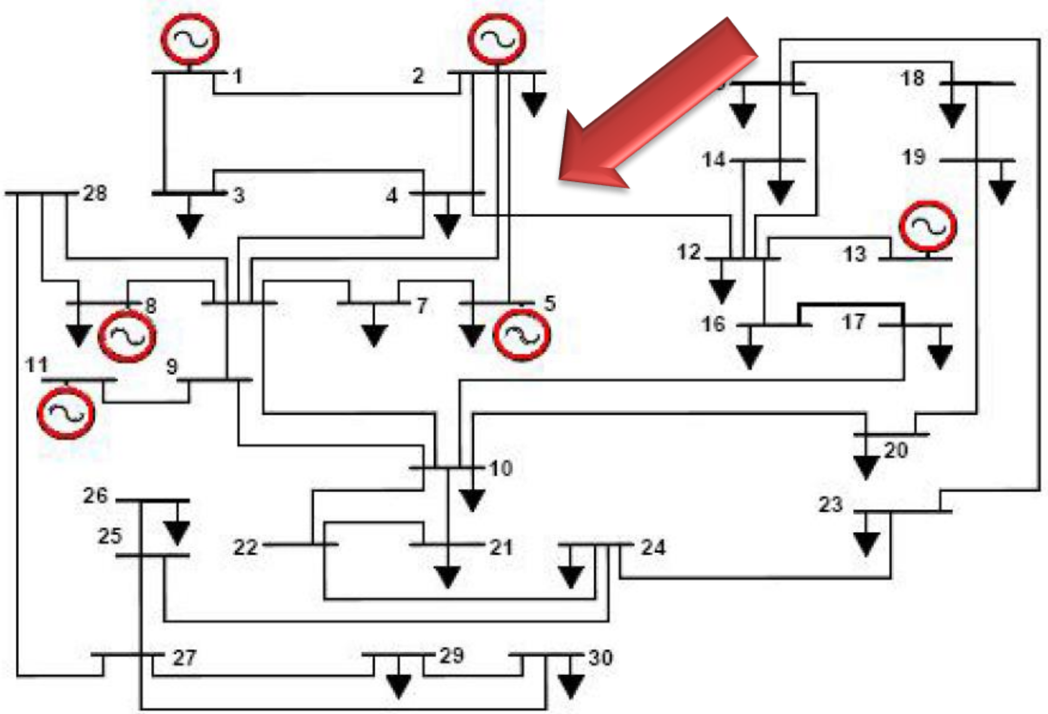
Overload Situation transmission line

Contingency on the line 2-5:

- Violation of generators : 0%;
- Violation of lines: Max 30%



Overload Situation transmission line

Dispatch:

Gerador	Perdas na Transmissão (Pt)	Custo na Geração (Cg)	Ponderação (Pt & Cg)
1	30.00	30.00	30.00
2	33.51	50.00	50.00
5	70.00	70.00	70.00
8	70.00	53.40	53.40
11	39.89	40.00	40.00
13	40.00	40.00	40.00
	18 iterações $F_{obj} = 82.37u.m$	17 iterações $F_{obj} = 20404.15u.m$	17 iterações $F_{obj} = 20484.50u.m$

Conclusions

- Predicting where overload will occur and their intensity;
- Operate with overloads safely;
- Restructuration Planning or System Expansion;

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Future Prospects

- Improvements to choice of parameters of barrier
- Studies on costs of violations;
- Predictor-Corrector method;
- Problems for Pre-Dispatch;
  - With maneuvers lines;
  - Constraints Ramp;


The End
Thank you for Attention!!!!